5.1 Area and Distance Problems

Learning Objectives: After completing this section, we should be able to

- find the area under a curve by estimating the sum of the areas of rectangular strips.
- apply finding the area under a curve in distance problems.

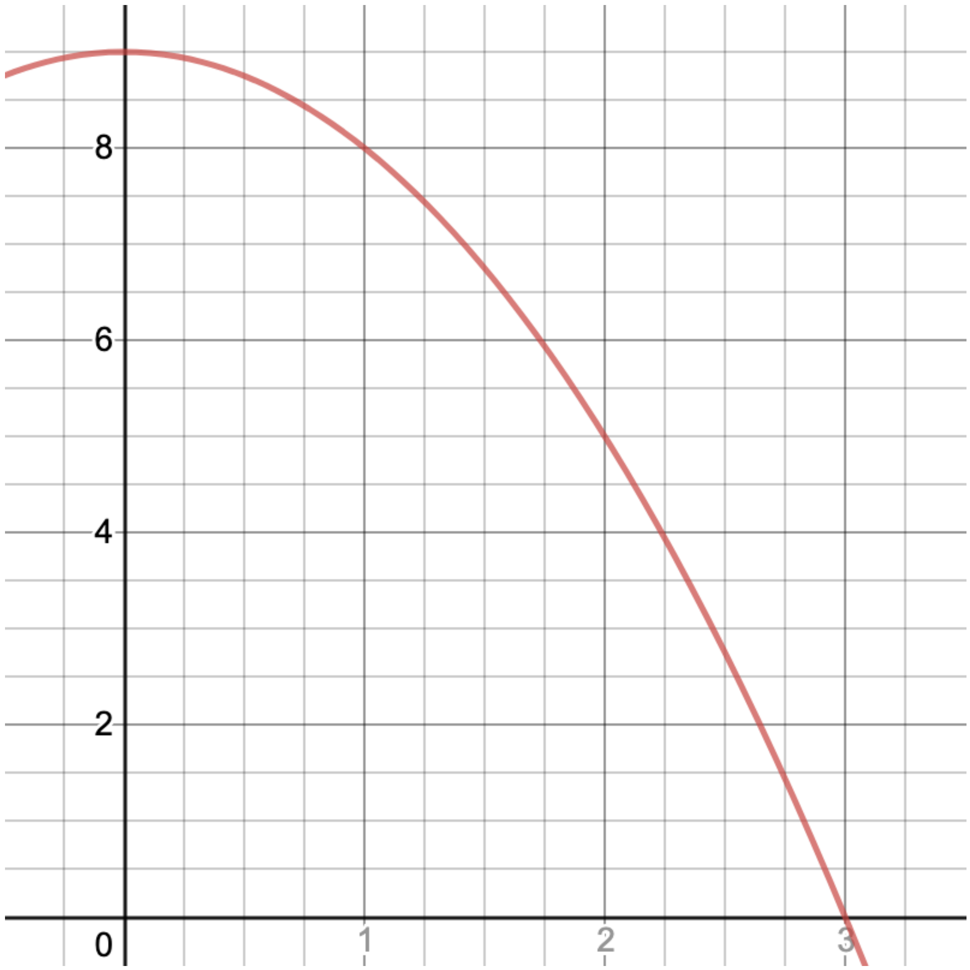
Example. Suppose I drove my car at a constant velocity of 55 MPH for 0.5 hours.

5.1.1 Area Under a Curve

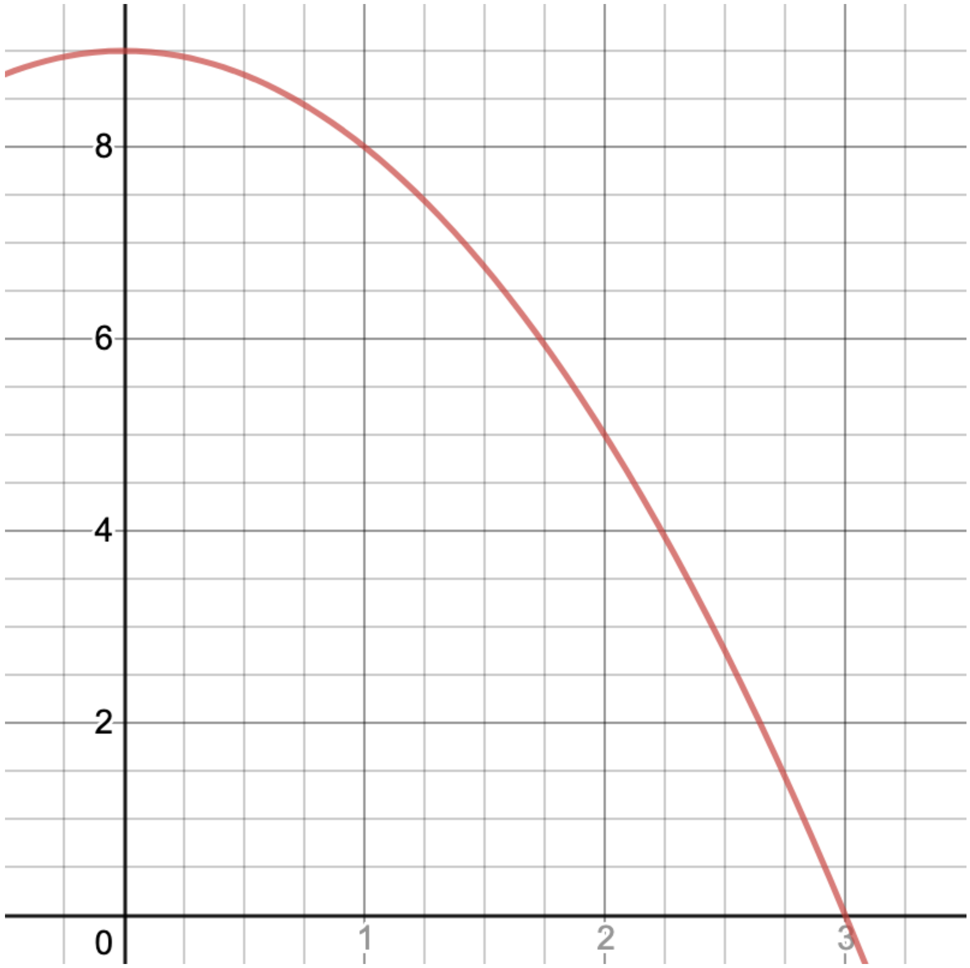
The area of the region under the graph of f on $[a, b]$ is

How can we approximate area under the curve?

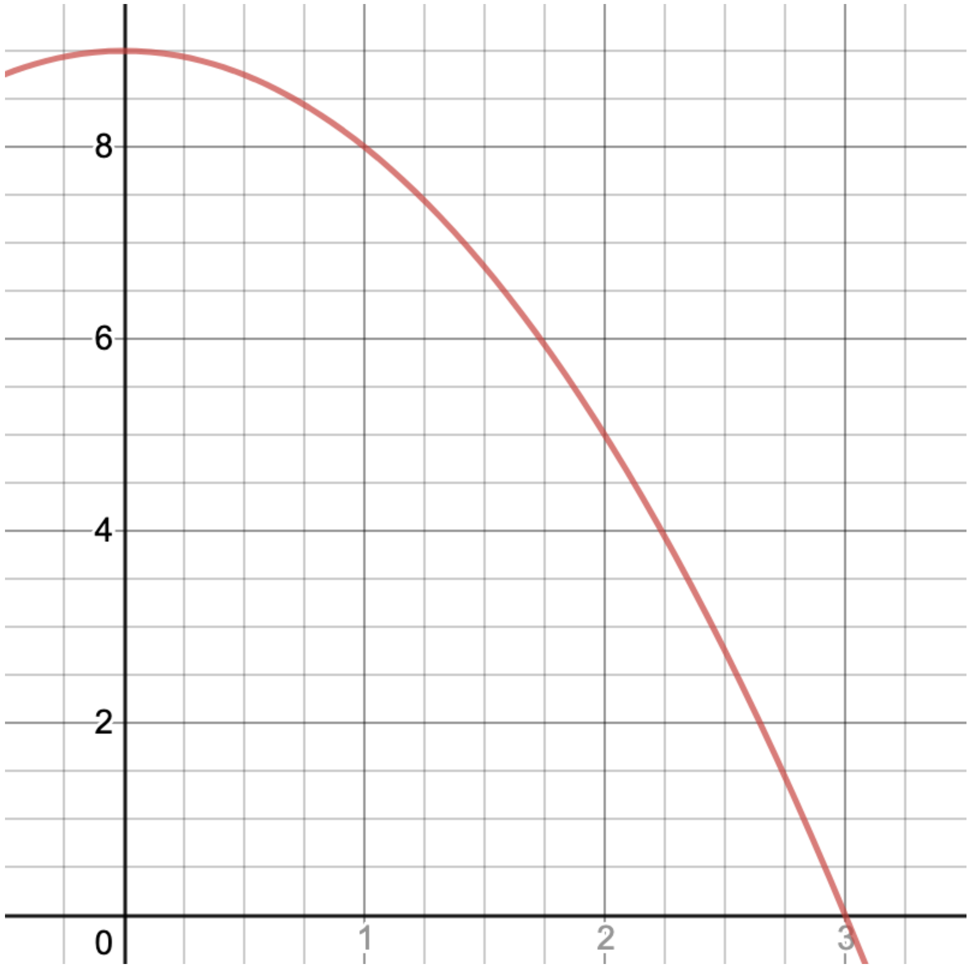
Example. Let $f(x) = 9 - x^2$ on the interval $[0, 3]$.



Let $f(x) = 9 - x^2$ on the interval $[0, 3]$.



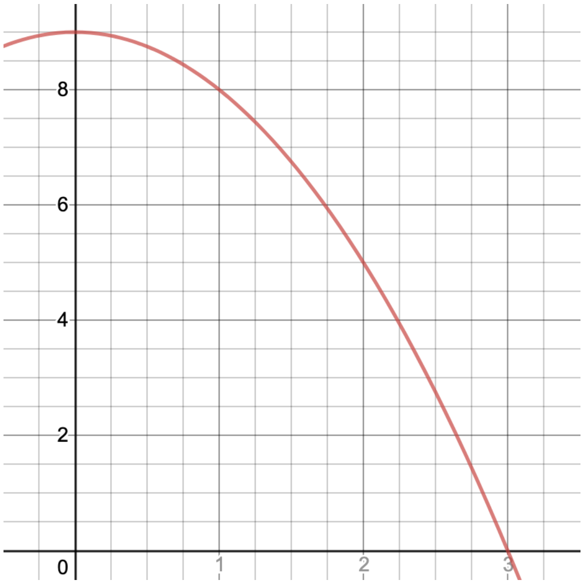
Let $f(x) = 9 - x^2$ on the interval $[0, 3]$.

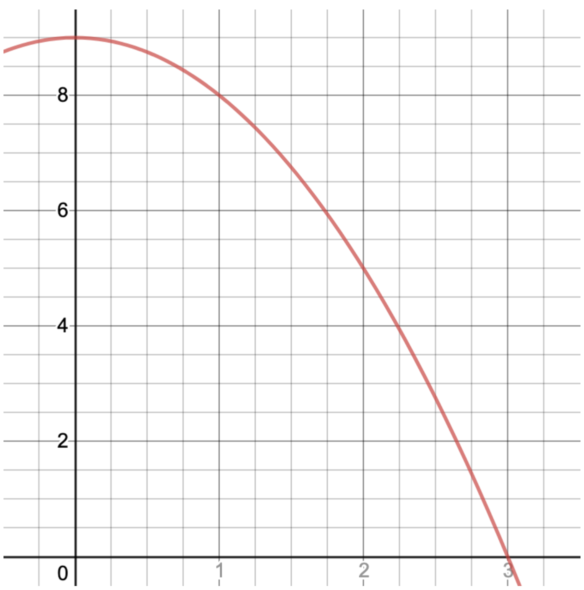


5.1.2 Exact Area

How can we improve our approximation?

Example. Let $f(x) = 9 - x^2$ on the interval $[0, 3]$.





5.2 The Definite Integral

Learning Objectives: After completing this section, we should be able to

- define a definite integral as the limit of a Riemann sum.
- evaluate definite integrals using summation properties.
- apply various properties of definite integrals.

Definition. Let f be a continuous function on $[a, b]$.

That is a lot to write, can we be lazier?

Definition. Definite Integral: Let f be a function defined on $[a, b]$.

5.2.1 Geometric Understanding of Area

Example. What does $\int_0^3 (9 - x^2)dx$ look like graphically?

Net Area under a Curve

So far, our examples have been positive.

Example.

We can estimate the area using *Riemann Sums*:
Suppose we have

- Left Riemann Sum

- Right Riemann Sum

- Midpoint Riemann Sum

Example. Approximate $\int_{-3}^{1} f(x)dx$ using $n = 4$ rectangles with a Left, Midpoint, and Right Riemman sums.

5.2.2 Properties of Definite Integrals

1. $\int_a^b f(x)dx + \int_b^c f(x)dx =$

2. $\int_a^b f(x)dx + \int_a^b g(x)dx =$

3. $\int_a^b f(x)dx =$

4. $\int_a^a f(x)dx =$

5. $\int_a^b cf(x)dx =$

6. $\int_a^b |f(x)|dx =$

Example. Suppose $\int_0^1 f(x)dx = 5$, $\int_0^2 f(x)dx = 2$, and $\int_1^2 g(x)dx = 3$. What is $\int_1^2 (2f(x) - g(x))dx$?

A note about dummy variables.

We can compute some definite integrals geometrically.

Example. $\int_{-2}^{3} |x-1|dx =$

Example. $\int_2^4 \sqrt{1-(t-3)^2}dt =$

5.3 Fundamental Theorem of Calculus

Learning Objectives: After completing this section, we should be able to

- establish the Fundamental Theorem of Calculus and apply it.
- identify the relationship between differentiation and integration as inverse processes.

5.3.1 Fundamental Theorem of Calculus, part 1

Recall: $\int_a^b f(x)dx =$

What if we write the function for area under $f(t)$ as

Let's find $\frac{dA}{dx} =$

Theorem. *If f is continuous on $[a, b]$,*

Example. $\frac{d}{dx}\int_1^x (t^3 + 1)dt$

5.3.2 Fundamental Theorem of Calculus, part 2

Theorem. *Let f be continuous on $[a, b]$. Then,*

Example. We found an approximate answer for $\int_0^3 (9 - x^2)dx$ in a previous section.

Example. Evaluate $\int_5^9 (7x^3 - e^x)dx$.

Example. Evaluate $\int_1^4 \left(\frac{3}{x^2} - 9\right) dx$.

You try!

Example. Evaluate $\int_1^3 \left(3x^2 - 8x + \frac{1}{x}\right) dx$.

You try!

Example. Evaluate $\int_{\frac{\pi}{2}}^{\pi} \cos(x) dx$.